Math 53, Discussion 105

Friday, April 3, 2020 9:46 AM

1. Describe what the surface $\phi = \frac{\pi}{3}$ looks like Cone
2. Evaluate $\iiint_E e^{\sqrt{x^2+y^2+z^2}} dV$ where E is enclosed by the sphere $x^2+y^2+z^2 = 9$ and the first octant
3. Set up the determinant to determine the integration constant for integrating in spherical coordinates
4. Find the Jacobian of the transformation $x = uv, y = \frac{u}{v}$
5. Compute $\iint_R x^2 dA$ where R is the region bounded by the ellipse $9x^2+4y^2 = 36$, (use the sub $x = 2u, y = 3v$)

1) Evaluate $\int_0^1 \int_0^1 e^{\max(x^2,y^2)} dy\,dx$

2) Find the average value of $f(x) = \int_x^1 \cos(t^2)\,dt$ on the interval [0,1]

3) Prove $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$ by the following:

A) using $\sum x^n = \frac{1}{1-x}$ show that $\int_0^1 \int_0^1 \frac{1}{1-xy} dx\,dy = \sum_1^\infty \frac{1}{n^2}$

B) evaluate the integral with sub $x = \frac{u-v}{\sqrt{2}}, y = \frac{u+v}{\sqrt{2}}$

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty \quad \Big| \quad \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty \quad = ???$$

3) $$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{if} \quad 0 \le x < 1$$

$$\int_0^1\int_0^1 \frac{1}{1-xy}\,dx\,dy = \lim_{t\to 1} \int_0^t\int_0^t \frac{1}{1-xy}\,dx\,dy = \lim_{t\to 1}\int_0^t\int_0^t \sum_{n=0}^{\infty} (xy)^n\,dx\,dy$$

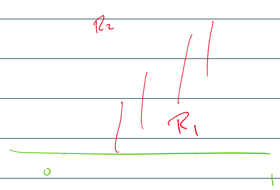
$$= \lim_{t\to 1} \sum_{n=0}^{\infty} \underbrace{\int_0^t\int_0^t x^n y^n\,dx\,dy}$$

$$\sum_{n=0}^{\infty} \lim_{t\to 1} \frac{t^{n+1}}{n+1}\,\frac{t^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{1}{(n+1)^2} = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\int_0^1\int_0^1 e^{\max(x^2,y^2)}\,dx\,dy$$

w is $x > y$

$x > y$



$$\parallel$$

$$\iint_{R_1} e^{x^2}\,dx\,dy + \iint_{R_2} e^{y^2}\,dx\,dy$$

$$\int_0^1\int_0^y e^{x^2}\,dy\,dx + \int_0^1\int_0^x e^{y^2}\,dx\,dy$$

$$= 2\int_0^1\int_0^y e^{x^2}\,dy\,dx = e-1$$

$\pi/2 \quad \pi/2 \quad 3$

1) $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{3} e^{\rho}\rho^2\sin\varphi \, d\rho\, d\varphi\, d\theta$

3) $x=\rho\sin\varphi\cos\theta$
$y=\rho\sin\varphi\sin\theta$
$z=\rho\cos\varphi$

$$\det\begin{pmatrix} \frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\theta} & \frac{\partial x}{\partial\varphi} \\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\theta} & \frac{\partial y}{\partial\varphi} \\ \frac{\partial z}{\partial\rho} & \frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial\varphi} \end{pmatrix} = \rho^2\sin\varphi$$

$\int e^x x^2\,dx$

x^2	e^x	
$2x$	e^x	+
2	e^x	−
0	e^x	+
0		−
0		

$x^2e^x - 2x e^x + 2e^x$

$\int u\,dv = uv - \int v\,du$

u	dv
du	v

dif ↓

a_0	b_0
a_1	b_1
a_2	b_2
a_3	b_3

↓ int

$a_0b_1 - a_1b_2 + a_2b_3 - \int a_3b_3$

$\int \log(x)$

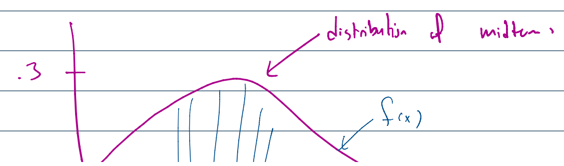
$\log(x)$	1
$\frac{1}{x}$	x

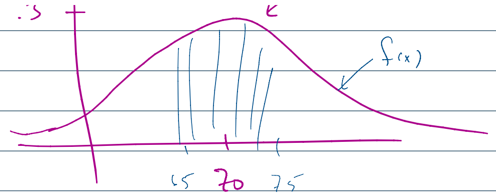
$\log(x)x - \int 1\,dx$

$\log(x)x - x$

in 1 variable, we have a probability density function (PDF), f.

given a random variable X, $P(a \le X \le b) = \int_a^b f(x)\,dx$





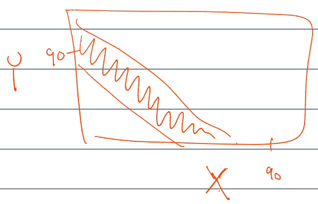
Probability of any student getting between 65 & 75 $= \int_{65}^{75} f(x)\,dx$

Now let X be a RV which is midterm 1 score
Y midterm 2 score

Define joint PDF $f(x,y)$ to be such that

Probability(X & Y are in a region R) $= \iint_R f(x,y)\,dx\,dy$

$\iint_{\mathbb{R}^2} f(x,y)\,dx\,dy = 1$



$60 \le \frac{X+Y}{2} \le 70$